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Consequences of the Dimension of the Quantum Function Ψ

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Abstract

Textbooks on classical physics adequately discuss the dimension concept of physical quantities. Alternatively, it turns out that quantum textbooks generally ignore the dimension of the quantum function ψ. Furthermore, different quantum field theories implicitly assign different values to this concept. This article discusses the dimension of the quantum function ψ of several theories. It proves that this concept yields effective criteria for the coherence of quantum field theories. These criteria show that the Dirac electron theory has the required properties. On the other hand, there are unsettled problems with the electroweak theory of the $W^{\pm}$ particles and with the Klein-Gordon theory of charged particles. The analysis shows a short proof of the inability to construct the required Maxwellian 4-current of the electroweak theory of the $W^{\pm}$ particles. This outcome indicates the effective properties of the dimension of the quantum function ψ. The discussion proves several other new constraints on the 4-current of a quantum theory of a charged particle.

Subject Areas

Numerical Mathematics, Quantum Mechanics

Keywords

Quantum Functions, Dimension, Coherence of Physical Theories, Elementary Charged Particles, Maxwellian Electrodynamics

1. Introduction

The concept of the dimension of a physical quantity appears very early in the general scientific education. For instance, the notion of length, area, and volume is probably known to pupils who have finished elementary school. The unit system is closely related to the concept of dimension, and the unit of meter (M) is an example of the unit of length. In this unit, length, area, and volume are measured by M, M^2, and M^3, respectively. Elementary physics textbooks explain the dimension concept of physical variables, and their index part mentions this issue (see e.g. [1]). This work analyzes the significance of the dimension of the quantum function ψ. An observation of the index part of many textbooks on quantum theory in general and quantum field theory (QFT) in particular indicates that they ignore this concept. This work proves that the dimension of a given quantum function ψ is an essential theoretical element, and this issue indicates the novelty of this work.

One can use several methods and easily find the dimension of the quantum function ψ of a given quantum theory. For example, consider the Schroedinger theory that defines the expectation value of a variable

$$\langle V \rangle = \int \psi^* V \psi \, \mathrm{d}^3 x \tag{1}$$

(see e.g. [2], p: 40). This expression proves that the dimension of the Schroedinger quantum function ψ is $[\mathrm{L}^{-3/2}]$, where [L] denotes the dimension of length. The present QFT structure depends on the least action principle, where the action is the appropriate integral of the theory's Lagrangian density. Here, the quantum function's dimension stems from the action's dimension and the specific form of the QFT Lagrangian density. One can also use the theory's structure and find the value of the dimension of its quantum function. The present work analyses the dimension of the quantum function ψ of several QFTs and proves the far-reaching consequences of this concept.

Expressions generally take the tensorial form of special relativity (SR). The discussion uses units where $\hbar = c = 1$. In this unit system, there is one type of dimension, and it is pointed out above that $[\mathrm{L}^n]$ denotes the power n of the length dimension. This work uses the standard notation, and the SR diagonal metric is $(1, -1, -1, -1)$. The second section briefly explains the significance of the Lagrangian density of a given QFT. The third section analyzes the role of the 4-current of an elementary charged particle. The fourth section discusses quantum electrodynamics (QED) of a charged Dirac particle. The fifth section discusses problems with theories of quantum function ψ whose dimension is $[\mathrm{L}^{-1}]$. The last section comprises concluding remarks.

2. The Significance of the Lagrangian Density

Today, the least action principle is a crucial QFT element. This action is the appropriate integral of the theory's Lagrangian/Lagrangian density. It yields the Euler-Lagrange equations that are regarded as the theory's equations of motion. For example, S. Weinberg states: "All field theories used in current theories of elementary particles have Lagrangians of this form" (see [3], p: 300). This work adopts this approach and examines several theories of elementary particles. The next discussion explains why the Noether theorem provides an excellent reason for the primary role of the least action principle.

The Noether Theorem

Conservation laws of physical quantities, such as energy, momentum, angular momentum, and charge, have acquired an unquestionable status in theoretical physics. The Noether theorem refers to the equations of motion of a closed system that the least action principle yields. It proves that if a given Lagrangian/Lagrangian density is invariant under a transformation of the space-time coordinates, then the system conserves energy, momentum, and angular momentum (see e.g. [3], section 7.3). Furthermore, if the Lagrangian density is a Lorentz scalar, then its equations of motion abide by SR (see e.g. [4], p: 35).

These requirements are satisfied simply by using a Lagrangian density that does not explicitly depend on the space-time coordinates, and its terms are Lorentz scalars. This practice means that any coherent theory that takes this form abides by the above-mentioned conservation laws. The present work adheres to this concept and examines the Lagrangian density of several QFTs.

Before entering into details, readers should note that the previous issues are *necessary conditions* for a coherent theory that is derived from the least action principle. However, these conditions are certainly not *sufficient* for an acceptable theory. For example, another meaningful condition says that a given physical theory should be free of mathematical contradictions.

The Noether theorem is significant in QFT theories that use the least action principle. A valuable application of the Noether theorem is the explicit expression for a conserved 4-current of a quantum particle

$$j^{\mu} = \frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \psi \tag{2}$$

Expression (2) says that a term of a given Lagrangian density that contains a derivative of the quantum function $\psi_{,\mu}$ yields a term of the 4-current (see [5], pp: 314-315). An arbitrary constant may multiply this expression. Generally, authors define this constant so that j^0 takes one of the following: particle's density, charge density, or mass density.

3. The Charged Particle's 4-Current

Consider the inhomogeneous Maxwell equation

$$F^{\mu\nu}_{,\nu} = -4\pi e j^{\mu} \tag{3}$$

This equation says that every theory of an elementary charged particle must provide a coherent expression for the 4-current j^{μ} (see [6], p: 85).

Here are several requirements that pertain to this issue:

1) The 4-current of a charged quantum particle must be a mathematically real 4-vector that depends on the quantum function ψ of the given particle.

2) The 4-current's dimension is $[L^{-3}]$.

3) The 4-current of a charged particle is used as a part of the electromagnetic interaction term of the particle's Lagrangian density. This interaction term affects the equations of motion of the charged particle and the Maxwell equations of the

electromagnetic fields (see e.g. [7], pp: 49-51, 78-89; [6], p: 85).

4) Due to the Noether expression for the 4-current (2), it must be free of derivatives of the quantum function $\psi_{,\mu}$ (otherwise, the electromagnetic interaction term destroys the original form of the 4-current it uses). The derivative-free requirement is vital because a later discussion proves that several physical texts ignore it.

The following analysis utilizes these requirements.

4. The QED Dirac Theory

QED uses the Dirac electron theory whose Lagrangian density is

$$\mathcal{L}_{Dirac} = \bar{\psi}\left[\gamma^{\mu}\left(i\partial_{\mu} - eA_{\mu}\right) - m\right]\psi \tag{4}$$

This Lagrangian density yields useful quantities (see e.g. [4], p: 78; [6], p: 84). The action

$$I = \int \mathcal{L}\mathrm{d}^4 x \tag{5}$$

has the dimension of $\hbar$, and in the unit system where $\hbar = 1$, it is dimensionless. Hence, the dimension of every Lagrangian density is [L^{-4}]. The [L^{-1}] dimension of mass and the Lagrangian density (4) prove that the dimension of the Dirac quantum function is [$L^{-3/2}$].

The Noether theorem (2) and the derivative term $\psi_{,\mu}$ of the Dirac Lagrangian density (4) yield the electron's 4-current (see [4], p: 50; [6], p: 97)

$$j^{\mu} = \bar{\psi}\gamma^{\mu}\psi. \tag{6}$$

Here, the γ^{μ} matrices are dimensionless. Moreover, the electron's Lagrangian density (4) proves that the dimension of the product $\bar{\psi}\psi$ is [L^{-3}]. Hence, (6) takes the required dimension of the 4-current. Note that the electromagnetic interaction term of (4) and the 4-current (6) are free of a derivative of the quantum function, and this 4-current satisfies all the requirements 1-4. This success has profound virtues, and here is how a textbook impressively describes the Dirac electron theory: "That such a simple Lagrangian can account for nearly all observed phenomena from macroscopic scales down to 10^{-13} cm is rather astonishing" (see [4], p: 78).

Conclusion: The previous analysis proves that the Dirac electron theory has a solid theoretical structure.

5. A QFT Whose Ψ Takes the [L^{-1}] Dimension

The present QFT literature describes several theories of elementary quantum particles whose dimension is [L^{-1}], such as the $W^{\pm}$, Z, Higgs, and Klein-Gordon (KG) particles. A general observation of these theories should pay attention to Weinberg's theoretical statement: "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear, and condensed matter physics" (see [3], p: 49). One can also find an explanation of this principle on pp. 1-6 of [8]. Hereafter, this relationship between QFT and quantum mechanics is called "Weinberg's correspondence principle". The term *good news* indicates the significance that Weinberg assigns to this concept.

Consider the standard form of the expectation value of a quantum particle (1). This expression depends on the spatial integral where the dimension of the quantum function is $[L^{-3/2}]$. It is unclear how a quantum theory of a particle whose ψ takes the dimension $[L^{-1}]$ can provide a coherent expression for the expectation value of a physical variable. Unfortunately, textbooks on the theories of these particles ignore this discrepancy. This is an example that shows how the violation of Weinberg's correspondence principle yields erroneous results.

The expectation value of a given variable is a crucial element of a quantum theory. Indeed, the quantum theory uses it to prove that it correctly describes the relevant data. It means that these theories violate the general physical requirement stating that an acceptable theory must describe relevant experimental data. This is a general contradiction of the theories of the $W^{\pm}$, Z, Higgs, and the Klein-Gordon particles. This unsettled contradiction justifies the rejection of these theories that belong to the standard model of particle physics (SM). A further discussion illuminates other problematic details of these theories.

This issue casts severe doubts about the correctness of SM glorifications of the current literature. Here are just two examples of these groundless declarations: "At various points in our discussion, we have noted that these theories have passed stringent quantitative experimental tests." ("these theories" == SM) (see [4], p: 781). "Remarkably, the Standard Model provides a successful description of all current experimental data and represents one of the triumphs of modern physics." (see [9], p: 1). Readers can find similar baseless declarations in the literature.

5.1. Problems with the $W^{\pm}$ Particles

The electrically charged $W^{\pm}$ particles are elements of the electroweak theory (see [4], p: 701). This theory is the SM weak interaction sector. The definition of its quantum function proves that its dimension is the same as that of the electromagnetic potential, namely, $[L^{-1}]$. The previous discussion and the electric charge of these particles indicate that, like the Dirac electron theory, the electroweak theory must provide a coherent expression for the $W^{\pm}$ 4-current. Unfortunately, searching for this expression in the electroweak textbooks is frustrating because one systematically fails to find it there. One may examine the following information and apply commonsense that indicates the veracity of this assertion.

Thousands of authors affiliated with CERN and Fermilab respectively, published the articles [10] [11]. CERN and Fermilab, are well-known physical research institutes. These articles discuss the electromagnetic interaction of the $W^{\pm}$ particles. It turns out that, unlike the Dirac electron theory, these authors admit that they do not use a Lagrangian but an "effective Lagrangian" for this interaction. The actual expressions that they use (see eq. (3) of [10] and eq. (1) of [11]) violate the requirements for a coherent 4-current. For example, they use the derivative of the quantum function of the $W^{\pm}_{\mu,\nu}$ particles. One may apply commonsense to this state of affairs and conclude that if thousands of physicists from respectable research centers such as CERN and Fermilab do not know a coherent expression for the 4-current of the $W^{\pm}$ particles, then nobody has published it before the publication time of these articles. The following proof of the inability to build a coherent 4-current for the electroweak theory of the $W^{\pm}$ particle justifies this assertion.

Let us examine the consequences of the [L^{-1}] dimension of the quantum function of the $W^{\pm}$ particles. The 4-current is a mathematically real quantity and its dimension is [L^{-3}]. One rejects the product of three functions of the $W^{\pm}$ because it is a mathematically complex quantity. The rejection of derivative of the $W^{\pm}$ stems from the corresponding Noether theorem (2).

Conclusion: The previous analysis proves that the electroweak theory of the $W^{\pm}$ particles has no coherent expression for the 4-current of these particles. This failure justifies the rejection of the electroweak theory.

This concise and clear proof illustrates the usefulness of the concept of the dimension of the quantum function ψ that is the main topic of this work. An obvious outcome of this issue is that the $W^{\pm}$ particles *are not elementary particles.* Hence, their integral spin proves that they must be mesons, namely quark-antiquark bound states, and the high mass of the $W^{\pm}$ indicates that one of these quarks is the top quark.

5.2. The Charged Klein-Gordon Particles

The Lagrangian density of a charged KG particle is (see [6], p: 38)

$$\mathcal{L}_{KG} = g^{\mu\nu}\phi^*_{,\mu}\phi_{,\nu} - m^2\phi^*\phi, \tag{7}$$

where ϕ is the mathematically complex quantum function of this KG particle. Using the [L^{-4}] dimension of the Lagrangian density and the product of the two derivatives as well as the quadratic mass term of (7), one finds that the dimension of the KG quantum function ϕ is [L^{-1}]. These are the same properties as those of the electroweak $W^{\pm}$ particles. Hence, the same contradictions pertain to the charged KG particle. For example, a textbook states that the 4-current of the charged KG particle is (see [6], p: 40)

$$j^{\mu}_{KG} = ig^{\mu\nu}\left(\phi^*\phi_{,\nu} - \phi\phi^*_{,\nu}\right). \tag{8}$$

This 4-current comprises a derivative of the quantum function $\phi_{,\mu}$, and the Noether theorem 4 of section 3, justifies its unacceptability.

6. Concluding Remarks

This work analyzes the dimension of the quantum function ψ. This is a new topic, and this paper proves that it has considerable scope that clarifies physical issues. For instance, it proves that the Dirac electron theory of QED has a coherent theoretical structure. On the other hand, it provides straightforward proof of the claims that the electroweak theory of the $W^{\pm}$ particles and the theory of a charged KG particle suffer from uncorrectable errors. The paper also proves an unnoticed aspect of Maxwellian electrodynamics, where the Noether theorem prohibits a 4-current that depends on the derivative of the quantum function $\psi_{,\mu}$. This is an unknown issue, and the literature indicates that thousands of authors are quite unaware of it. These virtues show the significance of this work. Moreover, it is interesting to point out that the refutation of the KG theory in this work agrees with Dirac's lifelong objection to the KG equation [12].

Conflicts of Interest

The author declares no conflicts of interest.

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